Mixed Strategy Nash Equilibria Notation.

For the following, assume there is a two player game. Player 1 is the row player, player 2 is the column player. S1 = {C,D}; S2 = {C,D}.

The **set** of all possible **strategy profiles** (which are combinations of S1 and S2) is therefore: S = {(C,C),(C,D),(D,C),(D,D)}.

Recall first of all that a Nash Equilibrium is a strategy profile. An example of a single strategy profile might be (C,D). (Note that the parentheses are different from curly brackets.) The set of Pure Strategy Nash Equilibria (PSNE) for a game might therefore be noted: PSNE = {(C,D),(D,C)}.
In other words, it is a *set* (indicated by the curly brackets "{") containing the *strategy profiles* (a particular selection of strategies, one for each player, denoted in parentheses) that are Nash Equilibria.

For notating a Mixed Strategy Nash Equilibrium (MSNE), we use the *mixed strategies* for each player, rather than the pure strategy for each player.

Assume that if Player 1 mixes between C and D with probability 1/2, and player 2 mixes between C and D with probability 1/3, 2/3, it is a Nash Equilibrium. In other words, we have a MSNE where sigma1 = (1/2, 1/2,) and sigma 2 = (1/3, 2/3).

There's a process for writing this out.
First, we start off with our "generic" MSNE answer:

$$MSNE = (\sigma_1, \sigma_2).$$

All MSNE with two players will take some variation of that form. This is not a complete answer, though; we still need indicate what sigma1 and sigma2 actually *are*. Fortunately, we know this from above: sigma1 = (1/2, 1/2) and sigma2 = (1/3, 2/3).

So, this brings us to step 2. In expanded form, we then write:

$$MSNE = (\sigma_1, \sigma_2)$$

$$\text{for } \sigma_1 = \left(\frac{1}{2}, \frac{1}{2}\right), \sigma_2 = \left(\frac{1}{3}, \frac{2}{3}\right).$$

For this MSNE, and for most that we actually encounter, we can then combine these two lines to make a simplified, correct answer. In other words, we put in the actual values for sigma1 and sigma2. **Note that we include the parentheses; otherwise the notation is not correct.**

$$MSNE = \left(\left(\frac{1}{2}, \frac{1}{2}\right), \left(\frac{1}{3}, \frac{2}{3}\right)\right).$$

That line would be a correct answer. And in general, we simplify whenever we possibly can. And to clarify, we only simplify if there is a single mixed strategy profile for each player and we *always* simplify if there is a single mixed strategy profile for each player in the MSNE. (So, you're able to put in actual numbers, and not just letters, for each player's mixed strategy.)

There are times when we cannot simplify. For example, if player 1 is indifferent between player C and D already, then there are an *infinite number* of strategy profiles that would make a MSNE; no matter what strategy profile player 2 plays, player 1 mixed strategy is still a best response.

In this case, player 1's mixed strategy would still be sigma1 = (1/2, 1/2). (This comes from how we solve MSNE. If you have questions, email me.)
But, player 2's mixed strategy sigma2 becomes more complicated. It would instead be:

$$\sigma_2 = (q, 1-q) \quad \text{for} \quad q \in [0, 1].$$

Note that the *entire line* above defines Player 2's mixed strategy. ("=", "for",... everything.)
Our MSNE would then become:

$$MSNE = (\sigma_1, \sigma_2)$$

$$\text{for } \sigma_1 = \left(\frac{1}{2}, \frac{1}{2}\right),$$

$$\sigma_2 = (q, 1-q) \quad \text{for } q \in [0, 1].$$

For the purposes of this class, we cannot simplify this any further. So, you would need to include all three lines in order to notate a MSNE where one player is already indifferent between two strategies, regardless of what the other player does.